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DESIGN AIDS AND SELECTION GUIDELINES FOR COMPOSITE HYPAR SHELL ROOFS WITH CUTOUTS- A FINITE ELEMENT APPROACH

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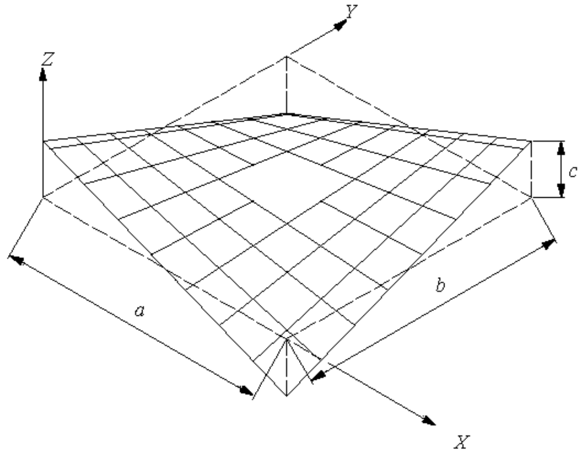
ABSTRACT
Composite hypar shell roofs with cutouts are analyzed for free vibration behavior using finite element method. Eight nodded curved quadratic isoparametric element is used to develop a computer code, and the formulation is validated through solution of benchmark problems which are earlier solved by other researchers. Numerical problems are solved for twelve different boundary conditions and four layered symmetric cross ply and angle ply laminated composite hypar shell to study the performance with respect to size of the cutouts and their positions in terms of fundamental frequency. Specific conclusions are drawn at the end, to summarize the outcome of the present investigation, which are expected to serve as important design aids to engineers engaged in composite shell construction.

Keywords: Hypar Shell, Composite, Finite Element, Free Vibration, Cutout.

1. INTRODUCTION
Different shell forms are commonly used in civil engineering applications. Among them which are used as roofing units, the skewed hypars (hyperbolic paraboloidal shells bounded by straight edges) have a special position because these architecturally pleasant forms may be cast and fabricated conveniently being doubly ruled surfaces. The hypar shells may be provided with cutouts for some service requirements. Shell forms used in civil engineering as roofing units have to withstand various distributions of dead and live loads apart from the dynamic loads from wind, earthquakes, snowfalls etc. Hence a comprehensive idea about their static and free vibration characteristics is essential for a designer for successfully applying these forms. Now-a-days researchers are emphasizing more on laminated composite shells realizing the strength and stiffness potentials of this advanced material. Application of hypars in industries often necessitates provision of cutouts for the passage of light, service lines and also sometimes for alteration of the resonant frequency.

The free vibration of composite as well as isotropic plate with cutout was studied by different researchers from time to time. But literatures on free vibration of composite shell with cutout are scanty. Chakravorty et al. [1] analyzed the effect of concentric cutout on different shell options. Again in 1999, Sivasubramonian et al.[2] studied the effect of curvature and cutouts on square panels with different boundary conditions. The size of the cutout (symmetrically located) as well as curvature of the panels was varied. Hota and Chakravorty [3] published useful information about free vibration of stiffened conoidal shell roofs with cutout. Later Nanda and Bandyopadhyay [4] studied the effect of different parametric variation on cylindrical shell with cutout.

A scrutiny of the literature that has accumulated on vibration of shell panels with cutout indicates that there is enough scope of research to be carried out about the hypar shell. Although Chakravorty et al. [1] did a few study about hypar shell with concentric cutout, many practically important aspects are yet to be addressed. In the present paper the author study the free vibration of hypar shell with cutouts (figure1) considering different boundary conditions. The variation of fundamental frequency due to change in eccentricity of cutout along x and y direction is also studied.



Surface equation: $z = \frac{4c}{ab}(x - a/2)(y - b/2)$

Fig 1. Surface of a hypar shell with cut out

2. MATHEMATICAL FORMULATION

An eight-noded curved quadratic isoparametric finite element (shown in figure 2) is used for hypar shell analysis. The five degrees of freedom taken into consideration at each node are u, v, w, α, β. The strain-displacement relations on the basis of improved first order approximation theory for thin shell are established as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial u/\partial y + \partial v/\partial x - 2w/R_{xy} \\ 0 \\ 0 \\ 0 \\ \alpha + \partial w/\partial x \\ \beta + \partial w/\partial y \end{Bmatrix} + z \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \partial \alpha/\partial x \\ \partial \beta/\partial y \\ \partial \alpha/\partial y + \partial \beta/\partial x \\ 0 \\ 0 \end{Bmatrix} \quad (1)$$

where ε and γ represent the direct and shear strains while *k* means the curvature.

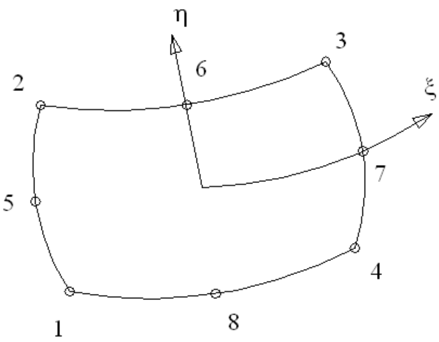


Fig 2. Eight noded shell element with iso-parametric coordinates

A laminated composite hypar shell (figure 3) of uniform thickness *h* and twist radius of curvature *Rxy* is considered. Keeping the total thickness same, the thickness may consist of any number of thin laminae each of which may be arbitrarily oriented at an angle θ with reference to the x-axis of the co-ordinate system. The constitutive equations for the shell are given by

$$\{F\}=[E]\{\varepsilon\} \quad (2)$$

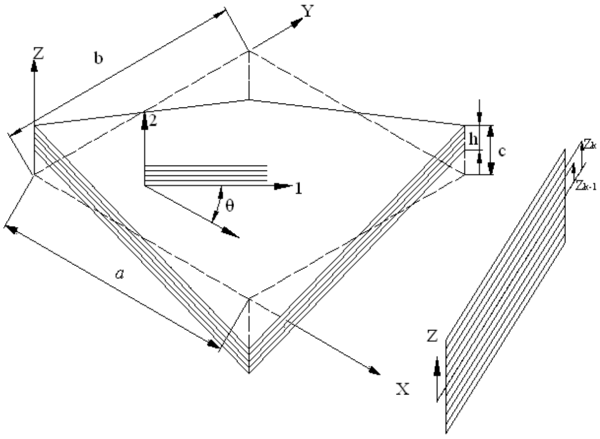


Fig 3. Laminations in skewed hypar shell

where,

$$\{F\} = \{N_x,\ N_y,\ N_{xy},\ M_x,\ M_y,\ M_{xy},\ Q_x,\ Q_y\}^T$$

$$[E] = \begin{bmatrix} [A] & [B] & [0] \\ [B] & [D] & [0] \\ [0] & [0] & [S] \end{bmatrix},$$

$$\{\varepsilon\} = \{\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy},\ k_x,\ k_y,\ k_{xy},\ \gamma_{xz},\ \gamma_{yz}\}^T \quad (3)$$

The stiffness coefficients are defined as

$$A_{ij} = \sum_{k=1}^{np} (Q_{ij})_k (z_k - z_{k-1});$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{np} (Q_{ij})_k (z_k^2 - z_{k-1}^2);$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{np} (Q_{ij})_k (z_k^3 - z_{k-1}^3) \quad \text{i,j=1,2,6;}$$

$$S_{ij} = \sum_{k=1}^{np} F_i F_j (G_{ij})_k (z_k - z_{k-1}) \quad \text{i,j=1,2} \quad (4)$$

where *Qij* are elements of the off-axis elastic constant matrix which are derived from appropriate transformation of the on-axis matrix. *Fi* and *Fj* are shear correction factors presently taken as unity. The terms of the on-axis matrix depend on the elastic moduli and Poisson's ratio of the material. The element stiffness matrix and consistent mass matrix are derived through the routine steps of finite element formulation employing numerical integration by using 2x2 Gauss quadrature. The element stiffness and mass matrices are assembled to get the global matrices [*K*] and [*M*].

The free vibration analysis involves determination of natural frequencies from the condition

$$\left| [K] - \omega^2 [M] \right| = 0 \quad (5)$$

This is a generalized eigenvalue problem and is solved by the subspace iteration algorithm.

2.1 Modelling the Cutout

The code developed can take the position and size of cutout as input. The program is capable of generating non uniform finite element mesh all over the shell surface. So the element size is gradually decreased near the cutout margins. One such typical mesh arrangement is shown in Fig. 4. Such finite element mesh is redefined in steps and a particular grid is chosen to obtain the fundamental frequency when the result does not improve by more than one percent on further refining. Convergence of results is ensured in all the problems taken up here.

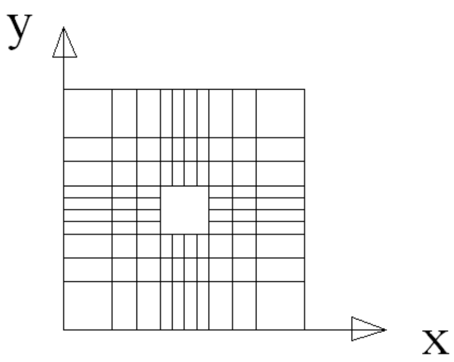


Fig 4. Typical 10 x 10 non-uniform mesh arrangements drawn to scale

3. NUMERICAL PROBLEMS

The accuracy of the present formulation is first established by comparing the results of the free vibration of simply supported and clamped hypar shell with $(0/90)_4$ shell of aspect ratio a/b=1, *c/a*=0.2 and side to thickness ratio a/h=100 with cutouts (Chakravorty et al. [1]). The non dimensional frequency parameter is $\bar{\omega} = \omega a^2 (\rho / E_{22} h^2)^{1/2}$. Material properties are as follows; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$. In order to study the effect of cutout size on the free vibration response additional problems for hypar shells with 0/90/0/90 and +45/-45/+45/-45 lamination and different boundary conditions have been solved. The boundary conditions are designated by describing the support clamped or simply supported as C or S taken in an anticlockwise order from the edge x=0. This means a shell with CCCC boundary is clamped along all four boundaries while a shell with CSCS boundary is clamped along x=0, simply supported along y=0 and clamped along x=a and simply supported along y=b, and so on. The shells considered are of square planform (a=b) and the cutouts are also taken to be square in plan ($a^{/}=b^{/}$). The cutouts placed concentrically on the shell surface. The cutout sizes (i.e., $a^{/}/a$) are varied from 0 to 0.4 and boundary conditions are varied along the four edges. The positions of the cutouts are varied along both of the plan directions of the shell for different practical boundary conditions to study the effect of eccentricity of cutout on the fundamental frequency.

4 RESULTS AND DISCUSSION

The fundamental frequencies of hypar shell with cutout obtained by the present method agree well with those reported by Chakravorty et al [1] as evident from Table1, establishing the correctness of the present results. The fact that the cutouts are properly modeled in the present formulation is thus also proved.

Table 2 furnishes the results of non-dimensional frequency ($\bar{\omega}$) of 0/90/0/90 and +45/-45/+45/-45 hypar shells. It is evident from Table 2 that in all the cases with the introduction of cutout with $a^{/}/a=0.1$ the increase or decrease in frequency is not more than 1%. But further increase in cutout size, i.e. when $a^{/}/a=0.2$, fundamental frequency decreases in all the cases, except for simply supported symmetric angle ply shells. Here also the decrease in fundamental frequency is between 1-10%. But further increase in cutout sizes $a^{/}/a=0.3$ and 0.4 fundamental frequency decreases to an appreciable extent (upto 30%). For some cases of symmetric angle ply shell no such unified trend is observed. This leads to the engineering conclusion that concentric cutouts may be provided safely on shell surfaces for functional requirements upto $a^{/}/a=0.2$.

The number of boundary constrains have a great impact than arrangement of boundary constraints. The fundamental frequency of a CCCC shell is highest. But with the introduction of simply supported edge one by one the fundamental frequency decreases. But keeping the number of boundary constraints same, the change in arrangement of boundary constraints do not change the fundamental frequency to an appreciable extent.

To study the effect of cutout positions on fundamental frequencies, results are obtained for different locations of a cutout with $a^{/}/a=0.2$. As with the introduction of cutout with $a^{/}/a=0.2$, the change in fundamental frequency with that of a un-punctured shell is within 1-10%, so $a^{/}/a=0.2$ is chosen for the further study. Each of the non=dimensional coordinates of the cutout centre ($\bar{x} = x/a, \bar{y} = y/a$) is varied from 0.2 to 0.8 along both the plan directions so that the distance of a cutout margin from the shell boundary is not less than one tenth of the plan dimension of the shell. The study is carried out for all the twelve boundary conditions for both 0/90/0/90 and +45/-45/+45/-45 hypar shells. The ratio of fundamental frequency of a un-punctured shell to that of a punctured shell is denoted by a ratio r. Table 3 and 4 furnishes the rectangular zones within which the centre of the cutout may be varied so that the value of r is always greater than or equal to 90-95. It is to be noted that the cutout centre may be placed at some points beyond the zones indicated in Table 3 and 4 to obtain similar values of r, but only zones rectangular in plan are identified in the tables. Table 3 and 4 also indicate the rectangular zone where r value is below 90. These tables will enable practicing engineers to get an idea of the maximum eccentricity of a cutout of which can be permitted if the fundamental frequency of an eccentrically punctured shell is not to suffer a drastic reduction in its value.

5. CONCLUSIONS

From the present study the following conclusions may be drawn.

(1) Concentric cutouts may be provided safely on shell surfaces for functional requirements upto $a^{/}/a=0.2$.

(2) The number of boundary constraint along the four

edges is far more important than their actual arrangement so far the free vibration stiffness is concerned

(3) Fundamental frequency undergoes marked improvement when the edge is converted to clamped from simply supported condition.

(4) Tables 3 and 4 provide a clear picture about the relative free vibration performances of shells for different combinations of edge conditions along the four sides and for different position of cutout and are expected to be very useful in decision making for practicing engineers.

6. REFERENCES

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7. NOMENCLATURE

Symbol	Meaning
a,b	length and width of shell in plan
$a^{/},b^{/}$	length and width of cutout in plan
c	rise of hypar shell
ε_x, ε_y	inplane strain component
γ_{xy} ,γ_{xz}, γ_{yz}	shearing strain components
G_{12},G_{13}, G_{23}	shear moduli of a lamina
np	number of plies in a laminate
z_k	distance of bottom of kth ply from mid-surface of a laminate
ν_{12}, ν_{21}	Poisson's ratios
ρ	density of material
ω	natural frequency
$\bar{\omega}$	non-dimensional natural frequency $=\omega a^2 (\rho / E_{22}h^2)^{1/2}$

8. MAILING ADDRESS

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Table 1: Non-dimensional fundamental frequencies ($\bar{\omega}$) for hypar shells (lamination $(0/90)_4$) with concentric cutouts

$a^{/}/a$	*Chakravorty et al. (1998)* Simply supported	clamped	*Present finite element model* Simply supported 8x8	10x10	12x12	clamped 8x8	10x10	12x12
0.0	50.829	111.600	50.573	50.821	50.825	111.445	111.592	111.612
0.1	50.769	110.166	50.679	50.758	50.779	109.987	110.057	110.233
0.2	50.434	105.464	50.323	50.421	50.400	105.265	105.444	105.443

a/b=1, a/h=100, $a^{/}/b^{/}$ =1, c/a=0.2; E_{11}/E_{22} = 25, G_{23} = 0.2E_{22}, G_{13} = G_{12} = 0.5E_{22}, ν_{12} =ν_{21} =0.25.

Table 2: Non-dimensional fundamental frequencies ($\bar{\omega}$) for hypar shells with concentric cutouts

Boundary conditions	*0/90/0/90* $a^{/}/a$ 0.0	0.1	0.2	0.3	0.4	*+45/-45/+45/-45* $a^{/}/a$ 0.0	0.1	0.2	0.3	0.4
CCCC	110.63	108.91	104.57	99.56	96.32	135.89	136.01	134.27	125.84	113.77
CSCC	105.38	104.44	101.31	89.61	81.55	119.47	120.83	112.18	93.68	84.54
CCSC	106.13	105.09	99.90	89.11	81.41	119.34	120.14	112.37	93.21	84.75
CCCS	105.38	104.43	100.83	89.14	81.40	119.04	119.94	112.17	92.90	84.26
CSSC	102.65	101.56	93.88	81.72	71.56	100.60	99.77	95.44	88.09	76.36
CCSS	102.75	101.61	94.29	81.73	71.65	100.59	99.62	95.38	87.75	76.26
CSCS	65.40	65.65	64.31	62.68	62.65	70.91	71.62	71.29	72.00	71.99
SCSC	65.05	65.31	63.94	62.27	62.17	71.23	71.90	71.61	71.23	72.57
CSSS	55.98	56.15	55.40	54.11	52.89	61.01	61.48	61.40	61.84	60.83
SSSC	55.94	56.11	55.34	54.02	52.74	61.16	61.61	61.55	61.98	61.12
SSCS	55.99	56.17	55.39	54.11	52.89	61.01	61.37	61.40	61.84	60.83
SSSS	48.59	48.70	48.29	47.18	45.64	53.77	54.01	54.21	54.67	53.97

a/b=1, a/h=100, $a^{/}/b^{/}$ =1, c/a=0.2; E_{11}/E_{22} = 25, G_{23} = 0.2E_{22}, G_{13} = G_{12} = 0.5E_{22}, ν_{12} =ν_{21} =0.25.

Table 3: Maximum values of r with corresponding coordinates of cutout centre and zones where r≥95, 90≤r<95,and r<90 for a 0/90/0/90 shell

Boundary conditions	***Maximum value of r***	***cutout centre ($\bar{x}$, $\bar{y}$) for max. values of r***	***Area in which r≥95***	***Area in which 90≤r<95***	***Area in which r<90***
CCCC	98.34	(0.2,0.2) (0.8,0.8)	$0.2\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.8$	nil	nil
CSCC	102.93	(0.2,0.8)	$0.2\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.3$	$0.3\le\bar{x}\le0.4$, $0.6\le\bar{x}\le0.7$ $0.4\le\bar{y}\le0.7$	nil
CCSC	97.17	(0.8,0.8)	$0.7\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.8$	$0.3\le\bar{x}\le0.6$ $0.3\le\bar{y}\le0.7$	nil
CCCS	97.28	(0.8,0.8)	$0.2\le\bar{x}\le0.8$ $0.7\le\bar{y}\le0.8$	$0.3\le\bar{x}\le0.4$, $0.6\le\bar{x}\le0.7$ $0.3\le\bar{y}\le0.6$	nil
CSSC	96.46	(0.8,0.8)	$0.2\le\bar{x}\le0.3$, $0.7\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.3$	$0.2\le\bar{x}\le0.6$ $0.4\le\bar{y}\le0.7$	nil
CCSS	96.31	(0.8,0.2)	$0.2\le\bar{x}\le0.3$, $0.7\le\bar{x}\le0.8$ $0.7\le\bar{y}\le0.8$	$0.2\le\bar{x}\le0.6$ $0.3\le\bar{y}\le0.5$	nil
CSCS	101.24	(0.3,0.5) (0.7,0.5)	$0.2\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.8$	nil	nil
SCSC	101.23	(0.5,0.3) (0.5,0.7)	$0.2\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.8$	nil	nil
CSSS	100.78	(0.5,0.3)	$0.2\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.8$	nil	nil
SSSC	100.74	(0.5,0.7)	$0.2\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.8$	nil	nil
SSCS	100.78	(0.7,0.5)	$0.2\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.8$	nil	nil
SSSS	99.59	(0.6,0.5)	$0.2\le\bar{x}\le0.8$ $0.2\le\bar{y}\le0.8$	nil	nil

a/b=1, a/h=100, $a^{/}/b^{/}$ =1, $a^{/}/a$ =0.2 , c/a=0.2; E_{11}/E_{22} = 25, G_{23} = 0.2E_{22}, G_{13} = G_{12} = 0.5E_{22}, $\nu_{12}=\nu_{21}$ =0.25.

Table 4: Maximum values of r with corresponding coordinates of cutout centre and zones where r≥95, 90≤r<95, and r<90 for a +45/-45/+45/-45 shell

Boundary conditions	***Maximum value of r***	***Cutout centre ($\bar{x}$, $\bar{y}$)for max. values of r***	***Area in which r≥95***	***Area in which 90≤r<95***	***Area in which r<90***
CCCC	102.56	(0.5,0.2)	$0.2\leq \bar{x} \leq 0.8$ $0.4\leq \bar{y} \leq 0.6$	No rectangular zone but some discrete points	nil
CSCC	104.29	(0.5,0.2)	$0.2\leq \bar{x} \leq 0.8$ $0.3\leq \bar{y} \leq 0.4$	$0.4\leq \bar{x} \leq 0.6$ $0.5\leq \bar{y} \leq 0.6$	$0.4\leq \bar{x} \leq 0.6$ $0.7\leq \bar{y} \leq 0.8$
CCSC	100.24	(0.5,0.2) (0.5,0.8)	$0.6\leq \bar{x} \leq 0.7$ $0.2\leq \bar{y} \leq 0.8$	$0.4\leq \bar{x} \leq 0.5$ $0.4\leq \bar{y} \leq 0.6$	$0.2\leq \bar{x} \leq 0.3$ $0.3\leq \bar{y} \leq 0.7$
CCCS	104.30	(0.5,0.8)	$0.2\leq \bar{x} \leq 0.8$ $0.6\leq \bar{y} \leq 0.8$	$0.4\leq \bar{x} \leq 0.6$ $0.4\leq \bar{y} \leq 05$	$0.3\leq \bar{x} \leq 0.7$ $0.2\leq \bar{y} \leq 0.3$
CSSC	100.96	(0.8,0.6)	$0.6\leq \bar{x} \leq 0.8$ $0.2\leq \bar{y} \leq 0.4$ $0.2\leq \bar{x} \leq 0.4$ $0.6\leq \bar{y} \leq 0.8$	$0.5\leq \bar{x} \leq 0.6$ $0.5\leq \bar{y} \leq 0.6$	$0.2\leq \bar{x} \leq 0.3$ $0.3\leq \bar{y} \leq 0.5$ $0.5\leq \bar{x} \leq 0.6$ $0.7\leq \bar{y} \leq 0.8$
CCSS	100.99	(0.8,0.4)	$0.2\leq \bar{x} \leq 0.4$ $0.2\leq \bar{y} \leq 0.4$	$0.5\leq \bar{x} \leq 0.6$ $0.4\leq \bar{y} \leq 0.5$	$0.2\leq \bar{x} \leq 0.3$ $0.5\leq \bar{y} \leq 0.6$ $0.5\leq \bar{x} \leq 0.6$ $0.2\leq \bar{y} \leq 0.3$
CSCS	101.79	(0.7,0.5)	$0.2\leq \bar{x} \leq 0.8$ $0.2\leq \bar{y} \leq 0.8$	nil	nil
SCSC	101.86	(0.5,0.3)	$0.3\leq \bar{x} \leq 0.7$ $0.2\leq \bar{y} \leq 0.8$	$\bar{x}=0.2,0.8$ $\bar{y}=0.5$	nil
CSSS	102.09	(0.7,0.5)	$0.2\leq \bar{x} \leq 0.8$ $0.2\leq \bar{y} \leq 0.8$	nil	nil
SSSC	102.14	(0.5,0.3)	$0.3\leq \bar{x} \leq 0.7$ $0.2\leq \bar{y} \leq 0.8$	$\bar{x}=0.2,0.8$ $0.4\leq \bar{y} \leq 0.5$	nil
SSCS	102.10	(0.3,0.5)	$0.2\leq \bar{x} \leq 0.8$ $0.3\leq \bar{y} \leq 0.7$	$0.4\leq \bar{x} \leq 0.5$ $0\ \bar{y}=0.2,0.8$	nil
SSSS	100.80	(0.5,0.5)	$0.3\leq \bar{x} \leq 0.7$ $0.2\leq \bar{y} \leq 0.8$	$\bar{x}=0.2,0.8$ $0.4\leq \bar{y} \leq 0.6$	nil

a/b=1, a/h=100, a'/b' =1, a'/a =0.2 , c/a=0.2; E_{11}/E_{22} = 25, G_{23} = 0.2E_{22}, G_{13} = G_{12} = 0.5E_{22}, $\nu_{12}=\nu_{21}$=0.25.